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Dynamics of plumes in a compressible mantle with phase changes: Implications for phase boundary topography

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A B S T R A C T

While plumes rising from the deep mantle may be responsible for hotspot volcanism, their existence has not yet been unambiguously confirmed by seismological studies. Several seismic studies reported that
the topography of the 670-km discontinuity is flat below hotspots, which disagrees with the elevation expected due to its negative Clapeyron slope and plume excess temperature. An improved numerical method that includes compressibility and consistently implemented phase transitions is used to study plume evolution in the Earth’s mantle. The influence of latent heat on plume behavior for varying convective vigor and Clapeyron slope of the endothermic phase change at 670 km depth is studied in axisymmetric spherical shell geometry. Minor differences in plume dynamics are found for models considering and neglecting latent heat. Three regimes of plume behavior at the endothermic phase boundary are observed: besides complete plume inhibition and penetration along the symmetry axis an intermediate regime in which the plume forms a ring around the symmetry axis is found.

These models also predict that the 670-km discontinuity is flat below hotspots due to a large plume head in the lower mantle of about 1000 km diameter that significantly thins as it rises into the upper mantle. This is explained by the lower viscosity in the upper mantle and the spreading of the temporarily inhibited plume below the endothermic phase boundary, which reconciles the flat 670-km discontinuity with a deep mantle plume origin.

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1. Introduction

The standard plate tectonic paradigm cannot explain long-lived hotspot volcanism, which occurs either far from tectonic plate boundaries (e.g., at Hawaii) or as excessive volcanism at plate boundaries (e.g., at Iceland). More than 60 hotspots and melting anomalies are observed on the Earth out of which at least 13 are characterized as hotspot chains with an age progression of more than 50 Myr (Ito and van Keken, 2007). Their occurrence may result from the ascent of hot material from the deep mantle in the form of mantle plumes (Morgan, 1971). It is under debate whether mantle plumes can be recognized in seismic images of the mantle. Though deep mantle low-velocity structures have been imaged below several hotspots (e.g., Ritsema and Allen, 2003; Zhao, 2004; Montelli et al., 2006; Wolfe et al., 2009) the anomalies differ in morphology and strength (e.g., Styles et al., 2011). Hwang et al. (2011) have shown that it is challenging to detect plumes in the lower mantle from traveltime delays due to wavefront healing. A combined approach of dynamical modeling and seismic imaging may help to understand whether seismic signatures can be linked to plumes.

Several mineral phase transitions are known in the Earth at which the mineral crystal structure changes (e.g., Ito and Takahashi, 1989). This has strong effects on the material properties and the dynamics of the rising hot plume material (Schubert et al., 1975; Christensen and Yuen, 1985). The first main (Mg,Fe)2SiO4 phase change from olivine (Ol) to Wadsleyite (Wad) occurs at approximately 400 km depth and is exothermic in nature. The second main phase change in this system is from Ringwoodite (Rw) to perovskite (Pv) plus magnesiowüstite (Mw) and occurs at approximately 670 km depth. Due to its endothermic nature this phase change is particularly important as increased temperature causes topography that counteracts the thermal buoyancy of rising plumes. This may enable layered convection (e.g., Christensen and Yuen, 1985) or at least decelerate up- and downwellings. The strength of the phase boundary elevation depends on the magnitude of the Clapeyron slope \( \gamma \). In the following \( \gamma (z) \) refers to the Clapeyron slope of the olivine system phase change at depth \( z \), where \( z \) is in km.

The phase transition from Wadsleyite to Ringwoodite occurs at approximately 520 km depth but both its density jump and the Clapeyron slope are not very well constrained and likely smaller in magnitude that those of the 400 and 670 km discontinuities (Yu et al., 2008). We neglect this transition in our study.
The Clapeyron slope is positive for Ol $\rightarrow$ Wad and negative for $R_W \rightarrow P_T + M_W$ (e.g., Ito and Takahashi, 1989; Ito et al., 1990; Bina and Helffrich, 1994; Katsura et al., 2003). The magnitude of $\gamma(z)$, however, is not precisely known, although most experiments indicate that the absolute value of $\gamma(400)$ is larger than that of $\gamma(670)$. Reported values are $2.9 \text{MPa/K} \leq \gamma(400) \leq 3.0 \text{MPa/K}$ (Bina and Helffrich, 1994) and $\gamma(400) = 4.0 \text{MPa/K}$ (Katsura et al., 2004) for the Ol $\rightarrow$ Wad phase transition and $-2.7 \text{MPa/K} \leq \gamma(670) \leq -1.9 \text{MPa/K}$ (Bina and Helffrich, 1994). $\gamma(670) = -(4 \pm 2) \text{MPa/K}$ (Ito et al., 1990) and $-2 \text{MPa/K} \leq \gamma(670) \leq -0.4 \text{MPa/K}$ (Katsura et al., 2003) for the $R_W \rightarrow P_T + M_W$ phase transition.

Latent heat effects during the phase changes also influence plume dynamics. At the $R_W \rightarrow P_T + M_W$ phase change latent heat is released inside an upwelling plume. This leads to an additional upward distortion of the phase boundary and additional negative buoyancy. Thermal expansion due to the increased temperature leads to a competing additional positive buoyancy (Schubert et al., 1975). Previous studies suggested that this might counteract the negative buoyancy effect of the elevated phase boundary for small $\gamma$ (Christensen and Yuen, 1985) which intriguingly causes more efficient convection than without phase changes.

The phase boundary distortions at 400 and 670 km depth that are predicted within a rising plume should be detectable with seismic methods. Some studies of P-wave conversions at the 400- and 670-km discontinuities indicate that the transition zone below hotspots is relatively thin (e.g., Shen et al., 1998a,b). Various regional studies have found a distinct depression of the 400-km discontinuity below hotspots, while the 670-km phase boundary lacks the distinct elevated topography expected from the Clapeyron slope and estimated plume excess temperature such as beneath the Afar region (Cornwell et al., 2011), the Pacific (Houser and Williams, 2010), Iceland (Du et al., 2006), the East African Rift (Huerta et al., 2009) and Hawaii (Courtier et al., 2007). We note that there is one important exception to this general trend. Below Yellowstone the 660-km discontinuity is elevated by more than 10 km while the 440-km discontinuity appears relatively flat (Schmandt et al., 2012).

Li et al. (2003) found that the transition zone is up to 15% thinner compared to IASP91 beneath various hotspots which is attributed mostly to the depression of the 400-km phase boundary. Tazuzin et al. (2008) report average transition zone thickness beneath 26 hotspots. However, they do observe a deepening of the 670-km phase boundary due to the 100–300 K higher temperatures but found that the 670-km phase boundary is not much affected.

Ritter et al. (2001) found a low-velocity anomaly beneath the Eifel extending to at least 400 km depth and suggested it to result from a deep plume (Goets et al., 1999) which is inhibited at 670 km. Grunewald et al. (2001) and Budweg et al. (2006) corroborate this with findings of a downwards deflected 400-km discontinuity and an unperturbed 670-km discontinuity. However, Lombardi et al. (2009) do not find a thin transition zone beneath the Eifel, which they relate to either the source of the central European hotspots being above the transition zone or insufficient resolution of their study.

While the observation of the flat 670-km phase boundary below hotspots may be explained by an upper mantle plume origin, several studies have reconciled it with a deep mantle origin. The lack of 670-km elevation in hot regions can be related to the garnet-system phase transition (majorite–perovskite) which is important between 1930–2020 K and has a positive Clapeyron slope (Houser and Williams, 2010), see also Deuss (2007), Tazuzin et al. (2008), Cornwell et al. (2011), Vinnik et al. (2012), Du et al. (2006), Huerta et al. (2009) and Courtier et al. (2007)). The lack of elevated topography below the Pacific and Africa may also be explained by large thermochemical piles that could extend up to 670 km depth (Lawrence and Shearer, 2008). Plume models that take into account a realistic reduction of the viscosity from the lower to the upper mantle (e.g., van Keken and Gable, 1995) predict significantly broader plumes in the lower mantle. It may therefore be possible that the 670-km discontinuity is elevated but does not show significant topography at the short wavelengths that can be resolved by the limited aperture seismological studies.

Phase changes are fundamentally related to mantle compressibility. While several whole-mantle convection studies considered mantle compressibility (e.g., Jarvis and McKenzie, 1980; Ito and King, 1994; Tackley, 1996), until recently most numerical simulations of mantle plumes assumed that the mantle can be described by an incompressible fluid under the Boussinesq (BA) or Extended Boussinesq Approximation (EBA) (e.g., Schubert et al., 1995; Kellogg and King, 1997; Marquart et al., 2000; Lin and van Keken, 2005; Lin and van Keken, 2006). Only few studies of plumes in a compressible mantle exist (e.g., Thompson and Tackley, 1998; Leng and Zhong, 2010; Leng and Gurnis, 2012) presented a compressible mantle plume model with a depth- and temperature-dependent viscosity and the Adams–Williamson equation of state as a reference state. Though Leng and Zhong (2010) implemented the 670-km endothermic phase change, their study neglected latent heat effects. Because of the competing buoyancy effects it is important to include phase changes consistently in fully compressible models and study their effects on mantle plume structure in the transition zone.

Here we present a numerical model of mantle plumes rising in a compressible mantle with phase changes, depth- and temperature-dependent viscosity and the Adams–Williamson equation of state as a reference state. Phase changes are implemented via a phase function in the Anelastic Liquid Approximation and latent heat effects are taken into account. We investigate the effect of latent heat on plume dynamics for a range of Clapeyron slope values of the $R_W \rightarrow P_T + M_W$ phase change. We are particularly interested in how plumes that have a buoyancy flux comparable to that observed at present-day hotspots (Davies, 1988; Sleep, 1990) interact with the transition zone and whether the predicted phase boundary deflections are consistent with seismological observations.

2. Model description and solution

2.1. Governing equations

We study thermal plume evolution in an axisymmetric spherical shell segment geometry. The equations governing the conservation of mass, momentum and energy are solved in the Anelastic Liquid Approximation (ALA) to account for mantle compressibility (Jarvis and McKenzie, 1980; King et al., 2010) as well as viscous dissipation and work done against gravity for an infinite Prandtl number fluid. These models extend the incompressible models introduced by Lin and van Keken (2005) and Lin and van Keken (2006).

The derivation of the equations with phase changes is provided in Appendix A. Here we summarize the main equations. Phase changes are introduced via the phase function $\Gamma$ which is chosen to have a hyperbolic tangent dependence on the non-dimensional excess pressure $\pi = \Gamma(\pi) = \frac{1}{2} (1 + \tanh (\pi/2))$ and $\pi = (p - p_\text{eq})/\rho g$ (Richter, 1973; Christensen and Yuen, 1985) with the phase transition pressure $p_\text{eq} = p_\text{eq} + \gamma T$. Here, $p$ is pressure, $\gamma$ is the Clapeyron slope, $T$ is temperature, and $d$ is the transition width. Reference values can be found in Table 1. Via the Gibbs–Duhem equation the Clausius–Clapeyron relation expresses the slope $\gamma$ of the equilibrium curve of two phases in the pressure–temperature space as a function of latent heat in dimensional form by
\[
\gamma = \frac{H_r \rho_1 \rho_2}{T (\rho_2 - \rho_1)} \tag{1}
\]

where \(\rho_1\) and \(\rho_2\) is the density above and below the phase boundary, respectively, and \(H_r\) is the latent heat per unit mass absorbed or released during the phase change (e.g., Schubert et al., 1975).

The mass conservation under the ALA is given by

\[
\nabla \cdot (\rho \mathbf{u}) = 0 \tag{2}
\]

where \(\rho\) is the reference profile of density and \(\mathbf{u}\) is the velocity.

The Stokes equation under the ALA including phase buoyancy effects in non-dimensional form is given by

\[
0 = -\nabla p' + \nabla \cdot \mathbf{r} + \left( \frac{c_p D r}{c_p D r} p' - \nabla R_a (T - T_i) + \frac{R_b}{\rho} (\Gamma - \Gamma) \right) \mathbf{g} \tag{3}
\]

where \(\mathbf{r}\) is the depth-dependent thermal expansivity, \(T_i\) is the reference temperature, \(\mathbf{r}\) is the reference phase function, \(\gamma_0\) is the Grüneisen parameter, \(\mathbf{r}\) is the deviatoric stress tensor, \(c_p\) is the specific heat at constant pressure, \(\mathbf{g}\) is the unit vector in the direction of gravitational acceleration, \(R\) is the adiabatic bulk modulus and \(p'\) is the dynamic pressure. For the purposes of this study we assume that \(c_p\) and \(\mathbf{r}\) are unity. The thermal and boundary Rayleigh numbers \(R_a\) and \(R_b\) are given by

\[
R_a = \frac{\rho_2^2 g x_i \Delta T h^2 c_p}{\kappa_i T_i} \tag{4}
\]

\[
R_b = \frac{\rho_2 \Delta \rho g h^2 c_p}{\kappa_i T_i} \tag{5}
\]

and the dissipation number \(D_i\) is defined as

\[
D_i = \frac{x_i g h}{c_p} \tag{6}
\]

where \(\eta, g, h\) and \(r\) refer to viscosity, gravitational acceleration and mantle thickness, respectively, and \(\Delta \rho = \rho_2 - \rho_1\). The subscript \(r\) refers to the reference value.

The heat equation that takes into account latent heating, viscous dissipation and work against gravity is given by

\[
\nabla \left[ \left( \frac{c_p}{\rho} \frac{R_b}{R_a} D_i \frac{dt}{dt} \frac{\Delta T}{\Delta T} + \frac{R_b}{\rho} (\Gamma - \Gamma) \right) \mathbf{g} \right] + \nabla \cdot (\Phi \nabla T) + \frac{D_i}{R_a} \Phi \tag{7}
\]

where \(\Phi, k\) and \(w\) are viscous dissipation, thermal conductivity, and vertical upward velocity, respectively. Reference values for relevant parameters are given in Table 1. \(R_b\) is the modified phase buoyancy Rayleigh number that is used in the latent heating terms in Eq. (7) and is defined as

\[
R_b = \frac{R_b}{\rho_2 \rho_1} \tag{8}
\]

We ignore internal heating. This is appropriate in this study since the long term effects of internal heating are included in the reference temperature.

The thermal expansivity \(\gamma\) in our model decreases with depth by \(\gamma = \gamma_0 (\rho / \rho_1)^{\delta}\) (van Keken, 2001). We assume \(\delta = 2\) which yields a reduction in \(\gamma\) of a factor of 3 from the surface to the core-mantle boundary.

The reference state is described by the Adams–Williamson equation of state with two phase changes. Thus the reference density profile with variable expansivity is

\[
\bar{\rho}(z) = \rho_i (\delta \frac{D_i z}{\rho_0} + 1)^{1/3} + \sum_{i=1}^{2} \Gamma_i \Delta \rho_i \tag{9}
\]

and the adiabatic temperature profile is

\[
\bar{T}(z) = T_i (\delta \frac{D_i z}{\rho_0} + 1)^{1/3} + \sum_{i=1}^{2} \Gamma_i \Delta T_i \tag{10}
\]

where \(z\) is the vertical coordinate and \(\Delta \rho_i\) and \(\Delta T_i\) denote the density- and temperature change across the phase transition \(i\). We use density changes of 5% and 10% of the reference density at 400 km and 670 km depth, respectively, which are similar to those of the phase boundaries in PREM (Dziewonski and Anderson, 1981). In all cases we numerically integrate the reference depth profiles for pressure, temperature, etc. and use these as look-up tables in the numerical models. The integrated mass of the mantle based on this reference density profile is \(4 \cdot 10^{24}\) kg.

We use a simplified viscosity profile

\[
\eta(T, z) = \eta(T) e^{-\alpha (T - T_i)} \tag{11}
\]

Table 1

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Reference parameter</th>
<th>Reference value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>Thermal expansivity</td>
<td>(3 \cdot 10^{-2}) K</td>
</tr>
<tr>
<td>(\gamma_0)</td>
<td>Grünerein-parameter</td>
<td>1.0</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Viscosity</td>
<td>(10^{12}) Pa s</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Density</td>
<td>(3300) kg/m³</td>
</tr>
<tr>
<td>(c_p)</td>
<td>Specific heat capacity at constant pressure</td>
<td>(1250) J/(kg K)</td>
</tr>
<tr>
<td>(d)</td>
<td>Phase transition width</td>
<td>(30) km</td>
</tr>
<tr>
<td>(D_i)</td>
<td>Dissipation number</td>
<td>(0.679)</td>
</tr>
<tr>
<td>(g)</td>
<td>Gravitational acceleration</td>
<td>(9.8) m/s²</td>
</tr>
<tr>
<td>(h)</td>
<td>Mantle thickness</td>
<td>(2878) km</td>
</tr>
<tr>
<td>(k_i)</td>
<td>Thermal conductivity</td>
<td>(10^{-6}) m²/s</td>
</tr>
<tr>
<td>(T_i)</td>
<td>Thermal expansivity</td>
<td>(1600) K</td>
</tr>
</tbody>
</table>

Plume evolution is studied for a viscosity reduction of 1/30 above 670 km depth and a temperature dependence expressed by \(b = \ln(10^2)\). In Section 3.3 we contrast the temperature- and depth-dependent viscosity models with those obtained with only depth-dependent viscosity \(\eta = \eta(z)\).

2.2. Numerical solution

We solve the governing equations using the finite-element code Sepron (Cuvelier et al., 1986). We use a form of dynamical mesh refinement that employs a staggered set of meshes that successively adjust in resolution to the rising plume head and tail to improve the efficiency and accuracy of the numerical model. A more detailed description of this method is provided in Appendix B. The boundary conditions are rigid on the top and side and free-slip at the bottom, which represents the core-mantle-boundary.

The initial temperature profile follows an adiabatic gradient in the mantle. Top and bottom thermal boundary layers (TBL) have a thickness of 96 km and 130 km, respectively. The temperature contrast is \(1327\) K across the top TBL and \(750\) K across the bottom TBL. The latent heat release and absorption at the two major phase transitions is \(1327\) K across the top TBL and \(750\) K across the bottom, which represents the core-mantle-boundary.

The latent heat release and absorption at the two major phase transitions is \(1327\) K across the top TBL and \(750\) K across the bottom, which represents the core-mantle-boundary. A harmonic (cosine) perturbation near the symmetry axis above the core-mantle boundary with an amplitude of half the temperature difference across the mantle and a non-dimensional wave number of 16 initiates the plume.


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3. Results

3.1. Plume evolution

3.1.1. Regime I: Plume ascent into the upper mantle

Fig. 2 shows the rising of a plume for temperature- and depth-dependent viscosity with two phase boundaries with \( \gamma(670) = -2.5 \text{ MPa/K} \) and \( Ra = 2 \cdot 10^6 \). As the plume reaches the transition zone it is initially impeded by the endothermic 670-km phase boundary, which is deflected upwards due to the increased temperature. However, the magnitude of \( \gamma(670) \) is not high enough to completely inhibit the plume. It penetrates the phase boundary along the symmetry axis and is then accelerated by the exothermic 400-km phase transition into the upper mantle. Finally the plume spreads below the lithosphere. Due to the low viscosity the plume conduit in the upper mantle is thinner than in the lower mantle. The primary plume head stays below the 670-km phase boundary and continues to feed the upper mantle plume. This behavior is observed for \( \gamma(670) > (-2.85 \pm 0.05) \text{ MPa/K} \). The uncertainty \( \pm 0.05 \) is the upper bound of half of the difference between two values of \( \gamma \) whose corresponding model results could clearly be identified with either Regime I, II or III.

The phase function \( \Gamma \) is shown in green in Fig. 2. While \( \gamma(400) \) is significantly deflected downwards during and after the transition of the plume, the topography of \( \gamma(670) \) flattens after plume transition.

We compute the buoyancy flux as described by Styles et al. (2011). The buoyancy flux in the upper mantle is on the order of

---

Fig. 2. Snapshots showing a plume evolution through the mantle for Regime I (\( \gamma(670) = -2.5 \text{ MPa/K} \)) with latent heat effects considered. The phase function for the two phase boundaries is shown in green. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)
15.6 Mg/s (at 500 km depth), which is slightly higher than that reported for the Hawaiian hotspot (Davies, 1988; Sleep, 1990; Asaadi et al., 2012).

3.1.2. Regime II: Intermediate plume regime

We find an intermediate regime of plume behavior for \((\gamma(670) > -3.05 \pm 0.05) \text{ MPa/K}\). Here the plume is stopped at the symmetry axis, but a secondary plume head erupts away from the axis. This causes a ring-shaped upwelling from the broad base of the plume at 670 km depth due to the symmetry assumed here (termed “ring-plume” in the following). A similar behavior was also found by Thompson and Tackley (1998). Tosi and Yuen (2011) find horizontal channel flow of hot plume material below the 670-km discontinuity that leads to plumes that are bent and whose upper-mantle conduit is thinner and offset of the lower-mantle conduit. They link the magnitude of the offset (or channel flow length) to the viscosity contrast. As the plume rises into the upper mantle and is accelerated by the exothermic 400-km phase change the ring closes starting from the bottom and eventually forms a cylindrical conduit in the upper mantle.

3.1.3. Regime III: Plume inhibition

The second endmember regime is the entire inhibition of the plume due to a higher magnitude Clapeyron slope \((\gamma(670) < 2.9 \pm 0.05) \text{ MPa/K}\). As the plume reaches the 670-km phase boundary, the phase boundary is elevated by the advection of heat but, due to the large magnitude of the Clapeyron slope, the plume spreads below the phase boundary and cannot rise into the upper mantle. The plume evolution through the lower mantle for Regimes II and III is the same as the typical head-and-tail shape shown in Fig. 2.

3.2. Plume behavior with and without latent heat effects

Since recent compressible plume models with phase changes have ignored the effects of latent heating (e.g., Leng and Zhong, 2010), it is of interest to study the influence of latent heat on plume dynamics. Fig. 5 shows the position of the top of the plume head.
with time for models that include and exclude latent heat for $Ra = 2 \cdot 10^6$. The comparison of the two cases shows only minor differences. Thus the influence of the phase boundary is similar irrespective of whether latent heat is considered, especially for Clapeyron slope values far from the critical value (curves for $\gamma(670) = 0$, $-2.2.5 - 3.2$ MPa/K in Fig. 5).

We would like to emphasize that it is essential to consider latent heat effects in the background state, too. If we assume a background state without latent heat effects, then adding these in the governing equations leads to a high discrepancy between the critical Clapeyron slope $\gamma_{crit}$ compared to the consistent models. The magnitude of $\gamma_{crit}$ is systematically larger by 2–3 MPa/K if latent heat is neglected in the background state but considered in the plume model compared to the consistent models (either latent heat considered consistently or completely neglected).

### 3.3. The dependence of the phase boundary influence for $\gamma(670)\rightarrow\gamma_{crit}$ on the Rayleigh number

We evaluate the influence of the endothermic phase boundary on plume dynamics for varying Rayleigh number. Fig. 6 shows $\gamma_{crit}$ as a function of Rayleigh number for (a) depth-dependent viscosity and (b) depth- and temperature-dependent viscosity. In both cases the magnitude of $\gamma_{crit}$ increases with decreasing Rayleigh number. $\gamma$ is related to the phase buoyancy parameter $P$ by $P = \gamma Rb/Ra$ (Christensen and Yuen, 1985).

For only depth-dependent viscosity the $\gamma_{crit}(Ra)$-dependence is investigated for models that either consider or neglect latent heat. When latent heat is ignored we find a slight shift of $\gamma_{crit}$ to higher magnitude values by $\Delta\gamma \approx 0.1$ MPa/K (Fig. 6(a)).

The error bars in $\gamma_{crit}$ in Fig. 6(a) denote the accuracy to which the models were tested for varying $\gamma$. Besides this they also represent uncertainties in determining $\gamma_{crit}$ due to instabilities in the lower-viscosity upper mantle, especially for slowly rising plumes ($Ra \leq 3 \cdot 10^6$). As $\gamma(670)\rightarrow\gamma_{crit}$, the plume conduit in the upper mantle becomes narrower and the time for which the plume is impeded below the phase boundary increases.

The maximum velocity in the conduit as a function of time shows two main pulses. Similar pulsating behavior was observed previously in both experimental and numerical studies (e.g., Vattekilde et al., 2009). These two pulses allow us to divide the plume interaction with the phase boundary into three regimes as well: the plume is either stopped, penetrates into the upper mantle, or the first pulse penetrates through the phase boundary while the second pulse is stopped. However, the small range of $\gamma$ for which this intermediate regime is observed and the uncertainties in $\gamma_{crit}$ make it difficult to accurately determine the separate curve for the intermediate regime.

The clear relationship between $\gamma_{crit}$ and $Ra$ shown in Fig. 6(a) is observed for only depth-dependent viscosity, but the trend is similar if temperature- and depth-dependent viscosity is assumed (Fig. 6(b)). The curve for depth-dependent viscosity with latent heat included in Fig. 6(b), the fitted curve follows $\gamma_{crit} = -0.31 \cdot e^{-Ra/2.5 \cdot 10^6} - 18.76 \cdot e^{-Ra/4.44 \cdot 10^6} - 0.23 \cdot e^{-Ra/2.5 \cdot 10^6} - (3.05 \pm 0.03)$ MPa/K.

The models with temperature- and depth-dependent viscosity can be separated into the three regimes identified in Section 3.1 (Fig. 6(b)). The overall trend of the curves separating the three regimes is comparable to that of Fig. 6(a), but for $3 \cdot 10^6 \leq Ra \leq 1 \cdot 10^7$ the critical Clapeyron slope for the transitions between the regimes is constant within the uncertainties. For this range of $Ra$ the intermediate regime is observed for a range of the Clapeyron slope of $(-2.95 \pm 0.05) \leq \gamma(670) \leq (-2.75 \pm 0.05)$ MPa/K. Over the range of Clapeyron slope values in Regime II the conduit of the plume evolving from the phase boundary progresses away from the symmetry axis. At the transition from Regime I to Regime II the plume passes the phase transition close to the symmetry axis and a cylindrical conduit is formed instantly which makes it difficult to determine $\gamma_{crit}$ dividing Regimes I and II more accurately than $\pm 0.05$ MPa/K.

Within the range of models we considered here we found for the models where the plume penetrates into the upper mantle (Regime I) that the temperature at 670 km depth ranges between 2000 and 2100 K, and that the temperature at 400 km depth is approximately 100 K lower. This indicates that due to the effects of compressibility we have a reduction in plume excess temperature from 750 K at the CMB to a more realistic upper mantle excess temperature of 250–300 K.

We tested the accuracy of our predictions by doing a ‘divergence test’ which involved computation of the plume models on
grids that had a lower resolution (with a maximum nodal point spacing of 4.50 km). This confirmed the trend of the curves in Fig. 6(a) with only a minor shift in $c_{\text{crit}}$ to lower magnitude values by $-2$ MPa/K. This suggests that, if anything, the curves in Fig. 6 may slightly underestimate the critical Clapeyron slope.

4. Discussion

4.1. Topography of the Rw $\rightarrow$ $P_\text{v}$ + $Mw$ phase boundary

Based on the estimated values of the Clapeyron slope a distinct depression of the 400 km and an elevation of the 670-km phase boundary is expected in a plume conduit. The 400 km depression is expected to be more distinct as its Clapeyron slope is estimated to be larger in magnitude than that of the 670-km phase boundary (Bina and Helffrich, 1994; Katsura et al., 2003, 2004).

While the 400-km phase boundary stays distinctly deflected downwards after plume transition in our models, the 670-km phase boundary shows a flattening that stays stable even after plume transition (Fig. 7 for moderately negative values of $\gamma(670)$). This flattening occurs naturally in our models as the primary plume head in the lower mantle is wider due to the higher viscosity of the lower mantle, combined with the inhibiting effect of the 670-km phase boundary. The high viscosity lower mantle alone is not sufficient to induce this pattern (Fig. 7(a)). These two effects cause the temperature below the 670-km phase boundary to be increased over a much broader, approximately ten times larger area than at the 400-km phase boundary, depending on the Clapeyron slope. The width of the uplifted region of the 670-km discontinuity increases with the magnitude of the Clapeyron slope and ranges from $\approx 800$ km for $\gamma = -2$ MPa/K to a maximum of 1300 km close to the critical Clapeyron slope. There is a moderate effect of Rayleigh number, due to the thinning of the plume in the lower mantle as $R_a$ increases. The range of widths of uplifted regions is generally higher than the seismic aperture used in most seismic studies. An important exception is in the interpretation of USArray data (Schmandt et al., 2012) where intriguingly a relatively narrow uplift of the 670-km discontinuity below Yellowstone is observed, indicating that the Yellowstone plume could have a significantly different shape than other plumes.

![](image.png)

**Fig. 6.** Critical Clapeyron slope $\gamma_{\text{crit}}$ as a function of Rayleigh number $R_a$ for (a) only depth-dependent and (b) depth- and temperature-dependent viscosity as defined in Section 2. The two curves in (a) describe models considering (blue) and neglecting (red) latent heat. The depth- and temperature-dependent viscosity case is for models considering latent heat. The three regimes of plume behavior (Figs. 2–4) are indicated in (b). In this model $\gamma_{\text{crit}}$ dividing the regimes is constant for $R_a > 2 \cdot 10^5$. The green curve in (b) represents the curve of the model considering latent heat in (a) with the green box indicating the range of values. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

![](image.png)

**Fig. 7.** Comparison of the final plume shapes for variable Clapeyron slope of the 670 km phase change. The combination of high lower mantle viscosity and the retarding effect of the phase change causes a flat topography of the 670 km boundary over a diameter of 1000 km. In contrast, the 400 km discontinuity has a pronounced short wavelength depression. This is similar to seismological observations and demonstrates that the absence of distinct 670 km phase boundary topography does not exclude a lower mantle origin of mantle plumes.
The broad and flat topography of the 670-km discontinuity predicted here provides a simple and elegant explanation for most of the seismic observations of 670-km phase boundary topography. This is independent of previously proposed explanations, such as the suggested domination of the exothermic majorite–perovskite transition at hot anomalies (Du et al., 2006; Courtier et al., 2007; Deuss, 2007; Tauxe et al., 2008; Huerta et al., 2009; Houser and Williams, 2010; Cornwell et al., 2011; Vinnik et al., 2012) or that large thermochronal piles may extend up to the transition zone beneath the Pacific and Africa (Lawrence and Shearer, 2008). In addition, the model results of the flat 670 km topography presented here are still compatible with a lower mantle origin of mantle plumes.

4.2. Plume behavior at the endothermic phase boundary

We have identified three regimes of plume behavior at an endothermic phase boundary for temperature- and depth dependent viscosity. The two endmember regimes are plume penetration along the symmetry axis (Regime I, Fig. 2) and complete inhibition of the plume at the phase boundary (Regime III, Fig. 4). In the intermediate regime the plume passes through the phase boundary away from the symmetry axis and thus forms a ring around the axis (Fig. 3).

The ring-plume that develops from hot plume material that spread below the 670-km phase boundary was observed previously by Thompson and Tackley (1998) for compressible convection with depth- and temperature-dependent rheology. A possible explanation may be that as the plume is inhibited and spreading below the phase boundary, it creates radial shear in the direction away from the symmetry axis, which leads to a downward velocity component at the axis above the 670-km phase boundary. The secondary ring-plume instability is then initiated away from the axis. For higher Clapeyron slope magnitudes the plume head is inhibited for a longer time and spreads more below away from the axis. For higher Clapeyron slope magnitudes the ring-plume instability is then initiated downward velocity component at the axis above the 670-km phase transition at hot anomalies (Du et al., 2006; Courtier et al., 2007; Deuss, 2007; Tauxe et al., 2008; Huerta et al., 2009; Houser and Williams, 2010; Cornwell et al., 2011; Vinnik et al., 2012) or that large thermochronal piles may extend up to the transition zone beneath the Pacific and Africa (Lawrence and Shearer, 2008). In addition, the model results of the flat 670 km topography presented here are still compatible with a lower mantle origin of mantle plumes.

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The ring-shaped conduit, however, is not stable. After penetration of the phase boundary the ring closes successively from bottom to top within a few Myr and forms a narrow conduit in the upper mantle.

While we found the formation of the ring plume consistently within the small parameter range indicated in Fig. 6, we recognize that this is likely a special occurrence that is partly induced by the assumed spherical shell symmetry. In three-dimensional geometry, especially with overriding plates, we expect that the ring plume will not rise symmetrically in the upper mantle and that it might even be suppressed more strongly, or form linear ridges into the upper mantle such as those observed by van Keken and Gable (1995).

The magnitude of the critical Clapeyron slope dividing the three regimes is dependent on the Rayleigh number (Fig. 6). The potential of the phase boundary to inhibit plume penetration increases with increasing convective vigor. This has been described earlier (Christensen and Yuen, 1985; Nakakuki et al., 1994) and is attributed to lower viscous coupling in the fluid at higher $Ra$.

The $\gamma_{\text{crit}}(Ra)$-dependence is more distinctive in the case with only depth-dependent viscosity, as only Regimes I and III are clearly observed in this case. The shift of $\gamma_{\text{crit}}$ for models considering versus neglecting latent heat may be attributed to competing effects at the phase boundary. These arise from the advection of heat (stabilizing) and latent heat release (stabilizing and destabilizing, as described in Section 1) (Schubert et al., 1975).

Due to the ring-plume regime observed for depth- and temperature dependent viscosity $\gamma$, cannot be determined as accurately as for the depth-dependent only viscosity case. Fig. 6(b) suggests that the general trend of $\gamma_{\text{crit}}(Ra)$ dividing Regimes I and II and $\gamma_{\text{crit}}(Ra)$ dividing Regimes II and III follows that of $\gamma_{\text{crit}}(Ra)$ for only depth-dependent viscosity. However $\gamma_{\text{crit}}(Ra)$ and $\gamma_{\text{crit}}(Ra)$ remain constant for $Ra \geq 3 \cdot 10^6$. For this range of Rayleigh numbers Regime II is observed for $(-2.95 \pm 0.05) \leq \gamma \leq (-2.75 \pm 0.05)$ MPa/K and the plume is completely inhibited for $\gamma < -2.95 \pm 0.05$ MPa/K. However, for $Ra < 3 \cdot 10^6$ which leads to plumes with a more realistic buoyancy flux, the magnitude of $\gamma_{\text{crit}}(Ra)$ and $\gamma_{\text{crit}}(Ra)$ increase with $Ra$. Whether or not this is a realistic range for the Clapeyron slope of the 670-km phase transition is not entirely clear. It agrees with the range of experimentally determined values for the Clapeyron slope of the 670-km phase transition of some studies (e.g., Ita et al., 1990), though other, more recent studies report a shallower slope (e.g., Bina and Helffrich, 1994; Katsura et al., 2003). Thus we consider it quite possible that the 670-km discontinuity can inhibit plume penetration in some cases.
4.3. Effect of a transition zone with high or low viscosity

The depth-dependence of the viscosity function that we used in these experiments is a simple, first-order approximation to the observed viscosity profile of the mantle, with a high-viscosity lithosphere and a jump in viscosity at the 670-km discontinuity. Several studies indicate significant more detail, including a viscosity maximum in the middle lower mantle and changes in viscosity between the uppermost mantle and the transition zone (e.g., King, 1995; Mitrovica and Forte, 2004; Soldati et al., 2009; Quinteros et al., 2010). Intriguingly enough we are still far from a consensus on whether the transition zone has lower or higher viscosity than the uppermost mantle, even within single studies that employ a particular approach (e.g., Steinberger and Calderwood, 2006). In almost all models the increase in viscosity at 670 km depth is a robust feature and our conclusions would not be affected significantly even if the uppermost mantle would have a higher viscosity than that of the transition zone (King, 1995; Soldati et al., 2009). We have investigated the consequences of assuming a viscosity profile where the transition zone has higher viscosity, by a factor of 10, than the uppermost mantle, while keeping the ratio between the viscosities of the uppermost mantle and lower mantle constant. We found small differences, such as the absence of regime II (i.e., ring plume formation is not observed), but the general conclusions regarding the values of the critical Clapeyron slope and relative flatness of the 670 km discontinuity are not affected.

5. Conclusions

We have presented a compressible plume model with consistently implemented phase transitions. We demonstrated that latent heat has little effect on plume dynamics at phase boundaries, but that it is essential to incorporate latent heat consistently into the background adiabat.

We identified three regimes of plume behavior at the endothermic phase boundary depending on the Clapeyron slope, with an intermediate ring-plume regime. This ring forms due to radial shear induced by the spreading plume, which causes secondary instabilities away from the symmetry axis.

We found that the 670-km phase boundary lacks a distinct elevation due to the wide plume head in the lower mantle and its spreading below the 670-km phase boundary. This observation due to the wide plume head in the lower mantle and its instabilities away from the symmetry axis.

We shear induced by the spreading plume, that causes secondary dissipation.

We found that the 670-km phase boundary lacks a distinct elevation due to the wide plume head in the lower mantle and its spreading below the 670-km phase boundary. This observation agrees with results of seismic studies of the mantle transition zone below hotspots and provides an alternative explanation for the observed flat 670-km discontinuity. The seismic observations can therefore not be used to argue conclusively against a lower mantle source for mantle plumes.

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Appendix A. Derivation of the conservation equations

The conservation equations for the Extended Boussinesq Approximation with one phase change presented by Christensen and Yuen (1985) are extended to the Anelastic Liquid Approximation for compressible flow.

The univariant phase change is an equilibrium curve in pressure−temperature-space with the Clapeyron slope

$$
\gamma = \frac{H_1 \rho_1 \rho_2 (\rho_2 - \rho_1)}{\rho_1 \rho_2} \quad \text{(A.1)}
$$

where $\rho_1$ and $\rho_2$ is the density above and below the phase boundary, respectively, $H_1$ the latent heat per unit mass absorbed or released during the phase change and $T$ the absolute temperature with $T = T^* + T'$. The overbar denotes the reference, only depth-dependent temperature-profile. Similarly, the total pressure is denoted by $p = \overline{p} + p$ with $\overline{p}$ the hydrostatic and $p'$ the dynamic pressure, and the density is $\rho = \overline{\rho}(\overline{p}, T^*) + \rho'$.

The phase boundary is described by the phase function $\Gamma$ which represents the relative fraction of the dense phase. Latent heat effects in the heat equation are then represented by the latent heat release per unit volume

$$
\rho h_0 \frac{\partial \Gamma}{\partial t} = \rho' T \frac{\partial \Gamma}{\partial x} - \rho_1 \rho_2 \frac{\partial T}{\partial x} - \rho_1 \rho_2 \frac{\partial T}{\partial y} - \rho_1 \rho_2 \frac{\partial T}{\partial z}
$$

(A.2)

with $R_g = \frac{\partial \rho}{\partial p}$ and $\Delta \rho = |\rho_2 - \rho_1|$. Following Christensen and Yuen (1985), the phase function $\Gamma$ is a function of the excess pressure $\pi = \overline{p} - \rho_1$ and $\gamma T = \overline{p}' T / \overline{p}$ is the phase transition pressure. The total derivative of $\Gamma$ is

$$
\frac{\partial \Gamma}{\partial t} = \frac{\partial \Gamma}{\partial x} \frac{\partial \Gamma}{\partial x} - \frac{\partial \Gamma}{\partial x} \frac{\partial \Gamma}{\partial x} \left( \gamma - \frac{\partial \Gamma}{\partial x} \right)
$$

(A.3)

where $\mathbf{u}$ is the velocity.

In the Anelastic Liquid Approximation changes of the reference density due to compression have to be taken into account, which thus affects the hydrostatic pressure gradient $\nabla \mathbf{p} = \overline{\rho g}$ where $g$ is the gravitational acceleration.

A.1. Heat equation

The heat equation is augmented by the latent heating term (A.2). With the work term simplified by ignoring terms depending on $p'$ and

$$
\frac{\partial \mathbf{p}}{\partial t} = \mathbf{u} \cdot \nabla \mathbf{p} = -c_p \mathbf{g}
$$

(A.5)

where $c_p$ is the heat capacity and assuming that $\rho' < \rho$ the dimensional heat equation becomes

$$
\overline{\rho c_p} \frac{\partial T}{\partial t} + 2 T \overline{c_p} g - \rho_1 \frac{\partial T}{\partial x} - \rho_1 \frac{\partial \mathbf{u}}{\partial x} - \rho_1 \frac{\partial \mathbf{u}}{\partial y} - \rho_1 \frac{\partial \mathbf{u}}{\partial z} = \nabla \cdot (k \nabla T) + \tau_1 \frac{\partial \mathbf{u}}{\partial x} + \rho H
$$

(A.6)

with $k$, $T$, $c_p$, $k$ and $H$ the thermal expansivity, the components of the stress tensor, specific heat at constant pressure, thermal conductivity and the internal heat production. $\tau_1 = \frac{\partial \mathbf{u}}{\partial x}$ is the viscous dissipation.

The equations are non-dimensionalized using the diffusion time scaling and the standard expression for thermal Rayleigh number $Ra$ (4), phase buoyancy Rayleigh number $Rb$ (5) and dissipation number $Di$ (6). In the following the starred symbols denote the non-dimensional parameter and the subscript $r$ denotes the respective reference value. The Clapeyron slope is non-dimensionalized by

$$
\gamma = \gamma' \frac{\rho gh}{\Delta T}
$$

(A.7)

where $h$ is the mantle thickness, which yields the latent heat term

$$
\frac{\rho_1}{\rho_2} \frac{\partial \Gamma}{\partial T} \left( \overline{p} + p' \right) \frac{\partial \Gamma}{\partial T} = \frac{\overline{p}}{\rho_1} \frac{\partial Rb}{\partial T} \frac{\partial \Gamma}{\partial T} + \frac{\partial \Gamma}{\partial T} \left( \overline{p} + p' \right) \frac{\partial \Gamma}{\partial T}
$$

(A.8)
where $R_b_1$ is the modified phase buoyancy Rayleigh number

$$R_b_1 = R_b \frac{\rho_2^2}{\rho_1^2 f_2}$$  \hspace{1cm} (A.9)

With this and now using the overbar for the non-dimensional reference state the non-dimensional heat equation becomes (7):

$$\bar{\rho} \left( \bar{c}_p + \frac{R_b_1}{Ra} \bar{D} \bar{T} + \frac{\bar{D}}{\bar{D} t} \right)\bar{D} + \bar{p} \left( \bar{c}_p + \bar{R}_b \frac{\bar{D}}{\bar{D} t} \right) \bar{D} \bar{r} \left( T + \frac{T}{\bar{D} r} \right) = \left( \nabla \cdot \left( \bar{k} \nabla T \right) + \bar{D} \bar{r} \Phi \right)$$  \hspace{1cm} (A.10)

This suggests the introduction of the modified expansivity and specific heat, as in Christensen and Yuen (1985), by

$$\chi' = \chi + \frac{R_b_1}{Ra} \frac{\bar{D} \bar{T}}{\bar{D} t}$$  \hspace{1cm} (A.11a)

$$C'_p = C_p + \frac{R_b_1}{Ra} \frac{\bar{D} \bar{T} + \bar{D}}{\bar{D} t}$$  \hspace{1cm} (A.11b)

A.2. Momentum equation

The dimensional momentum equation

$$- \nabla \bar{p} + \nabla \cdot \bar{T} + \bar{u} \bar{g} = 0$$  \hspace{1cm} (A.12)

becomes after non-dimensionalization and subtracting the hydrostatic pressure gradient

$$- \nabla \bar{p} + \nabla \cdot \bar{T} + \bar{u} \bar{g} \frac{\rho_1 c_p h^3}{K_s} \bar{z} = 0$$  \hspace{1cm} (A.13)

with $\eta$ the viscosity and $\bar{z}$ the unit vector in the direction of $\mathbf{g}$. The buoyancy term can be written as $\bar{f} \bar{z}$ with

$$\bar{f} = \frac{\bar{p} h \rho_1 g}{K_s} \left[ \frac{\nabla^2 \phi}{\nabla^2} + \frac{\rho_2^2 \nabla T \nabla h^3}{K_s} \left( \nabla \left( T - \frac{T}{K_s} \right) \right) + \frac{\Delta \rho}{\bar{p}} \frac{\rho_1 c_p h^3}{\eta_0} \left( \Gamma - \Gamma \right) \right]$$  \hspace{1cm} (A.14)

The pressure term can be expressed using the adiabatic bulk modulus $K_s$ and the Gruneisen parameter $\gamma_p$ and by further assuming $c_v = \text{const} = c_v_0$ as

$$\frac{\bar{p} h \rho_1 g}{K_s} = \bar{p} \frac{c_v h^3}{K_s} \frac{\bar{D} \bar{T} + \bar{D} \bar{r} (T - T)}{\bar{D} t}$$  \hspace{1cm} (A.15)

This leads to the non-dimensional momentum Eq. (3):

$$0 = - \nabla \bar{p} + \nabla \cdot \bar{T} + \frac{\bar{p} c_v h^3}{K_s} \frac{\bar{D} \bar{T} + \bar{D} \bar{r} (T - T)}{\bar{D} t} \frac{\bar{D} \bar{r}}{\bar{D} t}$$  \hspace{1cm} (A.16)

A.3. Reference state

The reference state for temperature is the adiabat $T$, for pressure the hydrostatic pressure $\bar{p}$ with $\nabla \bar{p} = \rho_1 \bar{g}$, and for the phase function $T = T_p \bar{p}$.

The constitutive equation for density is given by

$$\rho(\rho, \rho, T, \bar{T}) = \bar{p} \left( 1 - \chi(\rho, \rho, T) + \frac{\Delta \rho}{\bar{p}} (\Gamma - \Gamma) + \frac{\rho}{K_s} \right)$$  \hspace{1cm} (A.17)

where $K_s$ is the isothermal bulk modulus.

The reference state also depends on the assumptions for the phase change. The density is affected by the density contrast at the phase boundary and by $T$, while the adiabat is affected by latent heat release/absorption. This requires the integration of the density- and temperature gradients considering the modified thermal expansivity and heat capacity (A.11a, A.11b).

Appendix B. Evolving mesh refinement

We employ a staggered set of eleven meshes to improve the efficiency and accuracy of the numerical model. The resolution of these meshes is successively adjusted to the rising plume head and tail (Fig. B1). The highest nodal point resolution is 2.85 km.

The coarsening of the grid away from the thermal boundary layers and the plume leads to efficient solution of the equations but causes growth of numerical errors in the coarse regions, particularly below the upper thermal boundary layer and in the phase transition region. Since the temperature should remain adiabatic away from the plume and thermal boundary layers, we ignore the advective component in these regions by solving the heat equation assuming $u = 0$. This is employed in the coarse part of the mesh beyond 20° from the symmetry axis and in the upper mantle above 700 km depth, until the plume reached ~760 km depth. We investigate plume dynamics at various speeds by adjusting $Ra$. At low $Ra$ the plume rises sufficiently slowly so that the top boundary layer thickness significantly before the plume reaches the upper mantle. Thus we reduce the conductivity in the upper ~730 km to stabilize the top TBL until the plume reached ~760 km depth and then in the upper ~155 km. Additionally, for $Ra < 4 \times 10^5$ the top TBL was reset to its initial thickness with every interpolation on a new mesh until the plume reached ~760 km depth because of the slow rising of the plume.

References


