Convective mixing in the Earth’s mantle

Peter E. van Keken

Department of Geological Sciences, University of Michigan, 425 East University Avenue, 2534 CC Little Building, Ann Arbor MI 48109-1063, USA.
Phone: 1-734-764-1497, Fax: 1-734-763-4690, E-mail: keken@umich.edu

Chris J. Ballentine

Department of Earth Sciences, University of Manchester, Manchester M13 9PL, United Kingdom, E-mail: cballentine@fs1.ge.man.ac.uk

Erik H. Hauri

Department of Terrestrial Magnetism, Carnegie Institution of Washington, 5241 Broad Branch Road, NW, Washington DC 20015, E-mail: Hauri@dtm.ciw.edu

1. Introduction

The observed geochemical diversity of mid-oceanic ridge and ocean island basalts is generally attributed to the existence of large-scale and long-term chemical heterogeneity in the Earth’s silicate mantle. Yet, it is far from clear how this heterogeneity is generated and how it is maintained in the convecting mantle. Quantitative mixing studies indicate that the present day Earth is convecting vigorously enough to erase significant initial heterogeneity well within the age of the Earth. This suggests that some form of layering, or barrier against convective mixing is required to explain the geochemical observations. The most basic form of layering is that of the "classical" two-layer mantle, where a depleted and well-mixed upper mantle is separated by the seismically distinct 670 km boundary from a poorly mixed and enriched lower mantle. The mid-oceanic ridge basalts (MORB) originate from melting of the upper mantle, whereas ocean island basalts (OIB) derive from melting of material that is brought up from the lower mantle by plumes. This model has worked well to explain a large number of observations regarding noble gas and trace element concentrations and the distribution of mantle heat sources. However, in recent years this model has come under siege. Geophysical and geodynamical observations indicate that significant mass exchange occurs through the Earth’s transition zone. In addition, various geochemical observations suggest an important role for oceanic crust recycling in the plume source and demonstrate the lack of preservation of primitive mantle. The recent widespread acceptance of these fundamental problems of the classical layered mantle model has led the proposition of various alternatives such as the presence of layering below 670 km depth, or the preservation of heterogeneity in highly viscous regions in the Earth’s mantle. These models appear to be able to explain one or more features better than the classical model, but often cause new conflicts with existing geochemical or geophysical observations. In addition, it is not always clear that these
new conceptual models are physically realistic. With the advent of large scale computing, geodynamical modeling has become a particularly useful tool in this arena. Using the fundamental laws of the conservation of mass, energy, and momentum, models of mantle convection can be created that allow for quantitative tests of these conceptual models. Modeling also allows for a better understanding of the physics that governs mantle flow, mantle mixing and the distribution of chemical heterogeneity in planetary interiors.

This chapter will focus on the use of mantle convection modeling in the development of our understanding of the chemical evolution of the Earth by providing a short review of the main observations, a discussion of the physical approaches to characterize mantle mixing, and an overview of the historical and current modeling approaches to the formation, preservation, and destruction of chemical heterogeneity. Detailed reviews of the geochemical data and interpretations can be found in Zindler and Hart (1986), Silver et al. (1988), Carlson (1994), Hofmann (1997), Van Keken et al. (2002), Porcelli and Ballentine (2002), and Hauri (2002).

2. Geochemical and geophysical observations of mantle heterogeneity

2.1 Geochemical observations suggesting mantle layering

The formation of basalts by partial melting of the upper mantle at mid-oceanic ridges and hotspots provides the opportunity to determine mantle composition. Early studies of radiogenic isotopes in oceanic basalts (e.g., Faure and Hurley, 1963; Hart et al., 1973; Schilling, 1973) showed fundamental chemical differences between OIB and MORB. This led to the development of the layered mantle model, which consists essentially of three different reservoirs: the lower mantle, upper mantle and continental crust. The lower mantle is assumed primitive and identical to the bulk silicate Earth (BSE), which is the bulk Earth composition minus the core. The continental crust is formed by extraction of melt from the primitive upper mantle, which leaves the depleted upper mantle as third reservoir. In this model, MORB is derived from the depleted upper mantle, whereas OIB is formed from reservoirs derived by mixing of the MORB source with primitive mantle (e.g., DePaolo and Wasserburg, 1976; O’Nions et al., 1979; Allègre et al., 1979).

In particular the study of noble gas isotopes has provided a compelling case for mantle layering and the preservation of primitive mantle. For example, $^3$He/$^4$He values for MORB are nearly uniform, but large departures are seen for OIB. Most oceanic hotspots have elevated $^3$He/$^4$He values compared to MORB. $^4$He is generated by the decay of U and Th. $^3$He is not generated inside the Earth and is lost from the atmosphere upon degassing. The presence of high $^3$He/$^4$He values requires therefore the isolation of the plume reservoir from surface melting. It has been common to equate the high $^3$He/$^4$He ‘reservoir’ with the primitive mantle. Mass flux considerations add weight to the argument for mantle layering. For example, the inferred mass flux of $^4$He from the mantle is far lower than that predicted from the decay of U and Th in the whole mantle (O’Nions and Oxburgh, 1983), which suggests the existence of a boundary layer that decouples the flow of He and heat from the lower mantle. Steady state models for Pb and the noble gases similarly limit the mass exchange between upper and lower mantle (Galer and O’Nions, 1985; Kellogg and Wasserburg, 1990; O’Nions and Tolstikhin, 1994; Porcelli & Wasserburg, 1995ab).
Further suggestions for compositional stratification come from the consideration of the Earth’s heat budget. The combined heat production of the upper mantle and continental crust, based on estimates of radiogenic heat production of the crust and MORB-source (Rudnick and Fountain, 1995; Van Schmus, 1995) strongly suggests that radiogenic element concentrations in the deep mantle are significantly higher than that of the upper mantle.

2.2 Problems with the classical layered model

Several lines of evidence indicate problems with strict application of the layered model. It is particularly difficult to satisfy the requirement for primitive composition of the lower mantle. For example, Zindler and Hart (1986) showed that the isotopic database for the oceanic mantle could be described by mixing between depleted MORB mantle and enriched components that have compositions quite unlike primitive mantle. Similarly, the constancy of Nb/U and Ce/Pb (Hofmann, 1986; Newson et al., 1986) suggests that there is no current reservoir with primitive mantle ratios. Interestingly, there is a strong correlation between a depleted component, termed FOZO by Hart et al. (1992), and the high ³He source observed in many hotspots (Hart et al., 1992; Farley et al., 1992; Hanan and Graham, 1996).

In recent years, we have seen the use of new isotopic systems, including those of osmium and oxygen, in the description of oceanic basalts. Osmium isotopes can trace the addition of mafic crust or melt to the mantle source (Hauri and Hart, 1993) and have been used to highlight the presence of recycled mafic crust in sources of hotspots (Shirey and Walker, 1998). In a complimentary fashion, oxygen isotope variations trace directly portions of the mantle that have interacted with water at or near the Earth’s surface, which allows for identifying recycled oceanic crust in the sources of both OIB and MORB (Eiler et al., 1997, 2000; Lassiter & Hauri, 1998; Hauri, 2002). It has become apparent that nearly all hotspots display elevated ¹⁸⁷Os/¹⁸⁸Os, which requires involvement of mafic sources, most likely through the recycling of oceanic crust into the deep mantle. Mass balance calculations suggest the presence of a recycled oceanic crust reservoir of up to 10% of the mantle mass (Hauri and Hart, 1997). The strong evidence for the recycling of oceanic crust in the sources of hotspots is difficult to reconcile with the layered mantle unless most or all of the transition zone (which constitutes about 10% of the mass of the mantle) is composed of recycled oceanic crust and is somehow preferentially sampled by mantle plumes.

Geophysical observations are now increasingly used to argue against a form of layering at 670 km depth, in particular since it has been found that the sharp characteristics of the 670-km discontinuity (Paulssen, 1988) can be explained by a phase change in an otherwise isochemical mantle (Ito & Takahashi, 1989). Early reports of aseismic extensions of the slabs in the lower mantle based on travel time perturbations (Jordan, 1977; Creager and Jordan, 1984, 1986) and waveform complexity (Silver and Chan, 1986) suggested a dynamic connection between upper and lower mantle. Some of the first global tomographic models showed a pattern of high velocity in the deep mantle that correlated well with the inferred pattern of paleosubduction (Dziewonski, 1984; Dziewonski and Woodhouse, 1987). This was strengthened by a study by Lithgow-Bertelloni and Richards (1998), which suggested that tomographic models are explained
quite well if the lithosphere and lower mantle are one or two orders of magnitude more viscous than the upper mantle. This is in line with earlier estimates for the viscosity of the lower mantle based on geoid and gravity anomalies above subduction zones (Hager and Richards, 1989) and geodynamic inversions (Mitrovica and Forte, 1995). More indirect, but perhaps just as convincing, are arguments provided by Davies (1998) who shows that the lack of significant plume related topography at the Earth surface makes it inconceivable that a large thermal boundary layer exists at 670 km depth, as is required by the layered mantle hypothesis. Studies of the convective flow predicted by S-wave tomography also demonstrate that there is significant mass flux from the upper mantle into the lower mantle (Puster and Jordan, 1997). The quality of mantle tomography has increased dramatically in recent years, which has led to convincing images of slabs that are subducting into the lower mantle (Fukao, 1992; Grand, 1994; van der Hilst et al., 1997; Bijwaard et al., 1998; Ritsema et al., 1999; Grand, 2002). The correlation of seismically low velocities and subducted slabs is strengthened by the good agreement with paleographic reconstructions in several locations (Van der Voo et al., 1999).

In summary, we have recently witnessed a shift away from the classically layered mantle model in favor of whole mantle convection models, where the buoyancy of sinking slabs is the dominant driving force. Slabs can penetrate deep into the lower mantle and with the induced return flow we would expect the mantle to mix efficiently. This leaves us with an interesting dilemma. If the mantle convects as a whole, how can it preserve the large-scale and long-lived heterogeneity seen in the geochemistry of oceanic basalts?

3. Characterization of mixing

Quantitative evaluation of the efficiency of mantle mixing provides a fundamental tool to test conceptual models for the generation and destruction of heterogeneity. In this section, we will discuss and illustrate a number of ways in which we can quantify mantle mixing and provide a short review of the major findings from the literature. The physical processes of mixing in fluids are of great interest in a large number of fields, with important implications for industrial and engineering processes and a correspondingly large body of literature. For a comprehensive review of the mathematical description of mixing and examples of fluid dynamical and engineering applications, the reader is referred to Ottino (1989).

In this section we will provide some illustrations that are based on a simple two-dimensional convection model in Cartesian geometry. Full detail on the model setup, governing equations and numerical solution can be found in Appendix A. This model is strictly for illustrative purposes: it has a number of characteristics that we think are representative of convection in the mantle (infinite Prandtl number, finite dissipation number, temperature- and pressure-dependent viscosity) but falls short in a number of others (moderately low convective vigor, 2D Cartesian geometry, limited representation of rheology, no internal heating). A movie showing the convective motion is available at http://www.geo.lsa.umich.edu/~keken/treatise. The initial temperature field is shown in Figure 1a. The flow is characterized by moderate time-dependence of the boundary layers. The down- and upwelling that are near the center of the box remain at nearly stationary locations, but traveling instabilities along the top and bottom boundary layers
cause significant time-dependence in the center up- and downwelling. We will track two heterogeneities (numbered I and II) through this flow. The heterogeneities are identical in size and number of particles but differ in their initial position. Heterogeneity I is placed in the center upwelling, whereas heterogeneity II is placed just outside this upwelling (compare Figure 1a with Figure 1e). As we will show, the mixing behavior of these particle sets differs greatly; the heterogeneity that is started in the boundary layer encounters strong shear and reorientation and mixes with the fluid quite rapidly. Heterogeneity II remains inside a convective cell with only moderate deformation for a significant period of time. Similar extensive differences in mixing behavior within the same flow have been suggested in 3D models by Ferrachat and Ricard (1998) and Van Keken and Zhong (1999). Although we may predict that this ‘core-in-cell’ behavior (Spohn and Schubert, 1982) is not necessarily a fundamental explanation for the observed mantle heterogeneity, since mantle convection is significantly more time-dependent than this study, it may well form an explanation for observed regional differences, such as the DUPAL anomaly (Hart, 1984; Castillo, 1988).
Figure 1. a) temperature field of a moderately time-dependent 2D Cartesian convection calculation (see Appendix A for details). An initial heterogeneity (number I), discretized with 100x70 tracers, is placed in the upwelling near the center of the model. The heterogeneity is brought up to the surface and split in two sections. Arrows indicate the flow pattern that this heterogeneity will see. The numbers indicate the positions of the snapshots shown in the next frame. 1: t=0.001; 2: t=0.002; 3: t=0.0024; 4: t=0.0028; 5: t=0.0032.
b) deformation of heterogeneity I (left portion only) corresponding to the time intervals shown in frame a.
c) illustration of the definition of striation thickness.
d) illustration of the effects of increasing the number of tracers from 100x70 to 200x140, 400x280 and 800x560.
e) deformation of heterogeneity II that is slightly displaced from I.
3.1 Physics of mixing

Convective flows provide a mechanism for stirring and mixing of heterogeneities. The principle components of this process are stretching, folding, breakup, and diffusion (Ottino, 1989). Stretching and folding are illustrated in Figure 1. Stretching occurs in the presence of velocity gradients (such as a simple shear flow). Folding occurs when the material flow reverses onto itself or when the material encounters a fixed boundary (such as the core-mantle boundary and the free surface of the Earth’s mantle).

Other possible mixing mechanisms include diffusion and breakup. Breakup is negligible in the Earth’s mantle due to the lack of surface tension in solid rock. Diffusion in the solid phase is very slow due to the low compositional diffusion rates in mantle rocks, and is only important at very small length scales (and perhaps at very high temperatures like those at the CMB). The lack of interfacial tension and the similarity of properties between chemically distinct layers in the viscously deforming Earth allow us to consider the Earth’s mantle as a miscible fluid. We can therefore focus on stretching and folding as the primary mechanisms of mantle mixing. Nevertheless, the spatial and temporal discretization that is necessary in the numerical solution of the governing equations introduces breakup- and diffusion-like processes. Figure 1c shows the breakup of the heterogeneity due to the limited number of tracers that are used to represent the heterogeneity. This effect can be mitigated by introducing more tracers (see Figure 1d), which effectively leads to a lower sensitivity of the numerical model to artificial breakup. Similarly, the finite accuracy of numerical particle tracing leads to an effective numerical diffusivity (not illustrated).

Efficient mixing requires strong deformation. Since this occurs particularly in boundary layers, the heterogeneity of figure 1a was stretched and folded quite efficiently. It is interesting to illustrate the deformation of a heterogeneity that was initially positioned just outside the boundary layer (Figure 1e). The snapshots correspond to the same time-intervals as in Figure 1b. Due to the lack of strong velocity gradients in the core of the convection cell this heterogeneity is barely deformed at all. The dramatic difference in mixing efficiency between two different heterogeneities with slightly different initial condition is maintained over long computational time in this convection model, as is illustrated in Figure 2.

The processes of mixing may be conceptually simple, but the mathematical description is far from complete. The high viscosity of the Earth’s mantle compared to its thermal diffusivity implies that the inertial terms in the equations of conservation of momentum (the Navier-Stokes equations for general fluids) are negligible. The resulting Stokes equations predict laminar (non-turbulent) flow, which nevertheless can be quite complicated due to non-linear relationships between the equations of mass and heat transport and the highly temperature- and stress-dependent rheology of mantle rocks. The temporal distribution of heterogeneity in a fluid is strongly dependent on the mixing history. Small changes in the initial distribution can lead to exponentially growing differences, which means that even for simple flows we can witness chaotic or turbulent effects when observing mixing. This has been named Lagrangian turbulence in contrast to Eulerian turbulence, which describes the turbulent flow of material as for example seen in the airflow around airplane wings or the formation of thunderclouds. As discussed below, many studies find evidence for Lagrangian turbulence in mantle convection.
simulations. The turbulent nature of mixing makes a complete characterization of mixing impossible except for the simplest systems. For that reason it is essential to ask the question how we can best characterize mixing for any given situation.

Figure 2. Deformation of heterogeneities I (black) and II (gray) for extended model time.
In the remainder of this section we will discuss a variety of classical mixing indicators and their application to mixing in the Earth’s mantle. To introduce mixing concepts it is useful to start with a simple layered situation, where each of the layers has a constant thickness. Stretching in the direction of the layering will cause thinning of the layers; folding will cause duplication of the initial stacking. If we continue this process of folding and stretching we will witness an increasingly more complicated stacking of layers with decreasing thickness. Following the description by Ottino (1989) we can define simple measures of the efficiency of mixing such as the striation thickness, which can be defined as the average thickness of two neighboring layers (Figure 1c), or the intermaterial density area, which is the area of the interface per unit volume. Mixing will tend to reduce length scales by thinning and consequently reduce the striation thickness and increase the area of interfaces. In many cases, we see that length scales are reduced exponentially and that the interfacial area grows exponentially. It becomes impractical to describe the interface itself, but the exponential nature of mixing allows us to define typical time-scales of mixing as expressed for example by the increase in interfacial area. At a local scale we can predict that the distance \( dx \) between two particles, that were originally at distance \( dX \), will increase exponentially in efficiently mixing flows. It is convenient to define the infinitesimal stretching length \( \mu = dx/dX \) in the limit of \( dX \rightarrow 0 \). If the mixing is efficient the stretching length will increase exponentially, and \( d(ln \mu)/dt > 0 \).

In order to describe the mechanisms of mixing it is necessary to understand the local flow behavior. The motion around a point \( P \) can be approximated by

\[
V = v_p + dx (\nabla v) p + \text{higher order terms}
\]

where \( V \) is the velocity in a point that is a distance \( dx \) away from point \( P \), \( v_p \) is the velocity in point \( P \) and \( \nabla v \) is the velocity gradient tensor at point \( P \). The change in motion can be described by a stretch and a rotation using the stretching tensor \( D = \frac{1}{2}(\nabla v + (\nabla v)^T) \) and the spin or vorticity tensor \( \Omega = \frac{1}{2}(\nabla v - (\nabla v)^T) \). It can be shown that the mixing length and the stretching tensor are related by \( d(ln \mu)/dt \leq (D:D)^{1/2} \) where \( (D:D)^{1/2} \) is the second invariant of the stretching tensor which represents a basic measure of the magnitude of stretching. As a consequence, we can define the mixing efficiency as

\[
\varepsilon_\mu = \frac{d(ln \mu)/dt}{(D:D)^{1/2}}
\]

For incompressible flows it can be shown that the mixing efficiency has an upper bound of \( \{n-1)/n\}^{1/2} \) where \( n \) is the dimension of the space. It is instructive to display the mixing behavior of simple shear flow (Figure 3a). After an initial rapid increase of the efficiency, due to the stretching in simple shear flow, the theoretical 2D maximum of \( \frac{1}{2}\sqrt{2} \) is reached. After this the efficiency decays as \( 1/t \) since the fluid filaments become fully aligned with the flow. An efficient way to improve on the efficiency is therefore to regularly reorient the filaments so that they benefit from the early high efficiency. Such reorientation can take place at stagnation points in cellular convection or by time-dependent processes. Figure 3b shows the time evolution of mixing efficiency for a driven cavity flow, where deformation is dominated by shearing with occasional (‘weak’) redirection. This is quite similar to the case of 2D steady-state convection. In theory, a mixing process that combines simple shear flow with strong reorientation can retain high mixing efficiencies (figure 3c). It is also interesting to note that near fixed points (e.g., the corner regions of a single cell convection pattern) the velocity gradients are high. We can conclude therefore that the mixing efficiency is particularly high near fixed points.
Although the description of flow around these fixed points is based on a mathematical simplification, we can expect that the predicted mixing behavior is similar near areas of strong divergence (mid oceanic ridges, foundering of slabs at the core-mantle boundary) or converge (subduction zones, base of plumes).

![Figure 3. Illustration of the evolution of mixing efficiency for a) simple shear flow; b) simple shear flow with weak reorientation; c) shear flow with strong reorientation. Redrawn from Ottino (1989).](image)

### 3.2 Tools for the study of mantle mixing

Fluid dynamical studies of mixing can be roughly separated into three different categories based on the approach that is taken. Analytical studies allow generally for a continuous mathematical description of the physics. Although they are generally limited to only the simplest mixing geometries, a large number of our descriptive tools are derived from these analytical studies. Experimental mixing studies are common in many engineering applications and in studies where the behavior of laboratory fluids can be used to approximate large-scale processes. For planetary convection applications it is essential to make extrapolations to the much different length and time scales than those of the laboratory. An important benefit of the experimental approach is that physical processes can be studied with much higher resolution than is possible using numerical methods. Numerical approaches, which solve the governing equations of conservation of mass and momentum in discrete form, can suffer from discretization errors in time and space, although convergence tests can be used to determine whether these errors have a great impact on the solution to the problem that is being investigated. An important benefit of numerical approaches is that the equations can be solved with the correct spatial and temporal dimensions without the need to extrapolate from laboratory conditions, and that data on the properties of the entire fluid can be recorded throughout the convection experiment.

Of particular interest for mantle mixing is the dispersion of heterogeneity (e.g., the efficiency of mixing of sediments and oceanic crust upon subduction into the mantle) and many studies approach mantle mixing particularly from this perspective. Most approaches we will discuss here follow from numerical convection experiments that use concepts derived from analytical and experimental studies. We will introduce and
illustrate the main approaches in a heuristic manner. For a more rigorous approach see Ottino (1989) and references therein.

Tools that have been used to visualize the mixing behavior of a fluid include:

1. Streamlines that provide an instantaneous picture of the velocity field. In stationary flows the streamlines represent particle paths. Note that the term ‘stationary’ in fluid dynamical literature means ‘steady-state’ and not ‘static’. In other words, a stationary velocity field is one in which fluid moves, but the pattern or speed doesn’t change.

![Streamlines](image)

*Figure 4. Streamlines corresponding to the snapshots in Figure 2. Contour interval is 50 (non-dimensional) units. Note the focusing of the flow in the center upwelling due to the temperature-dependence of the viscosity.*
2. Particle paths (also called orbits or trajectories) that follow the motion of individual particles as they are advected by the velocity field. These can be particularly insightful to illustrate regions in the flow that differ in mixing behavior (figure 5).

![Particle paths](image.png)

Figure 5. Particle paths corresponding to the convective mixing of the model shown in Figure 1 and 2. Each particle is followed from \( t=0 \) to \( t=0.0109 \). The initial position is indicated with the solid rectangles. Arrows illustrate the direction of the particle motion. The gray particle paths correspond to the efficiently mixed heterogeneity I (figure 1d); the black particle paths correspond to the poorly mixing heterogeneity II (Figure 1e). Note that the gray particles traverse only about 2 convection circulation, but build up significant deformation in this time period (compare Figure 2).

3. Plots representing the time evolution of a particle cloud representing an initially small heterogeneity. This allows for a visual appreciation of the nature of mixing and at least for qualitative estimates of the spatial and temporal scales of mixing (Figures 1 and 2).

4. Poincaré maps allow for a descriptive simplification by a reduction of the dimension of space. An often-used application of this general mathematical mapping is to plot the intersection of particle paths with a fixed surface. These maps are useful to identify regions of different mixing properties (e.g., laminar versus chaotic) in steady-state flows and identify temporal changes in periodic or time-dependent flows (Figure 6).
Figure 6. Poincaré plots representing the time and horizontal position of particles traveling through the surface $y=0.5$. a) for 21x15 particles that are distributed in heterogeneity I. b) same, but now for heterogeneity II. The general structure of the flow, the reasonably efficient mixing of a) compared to the more periodic and poor mixing of b) are clearly illustrated. Note the transition to apparently more efficient mixing after $t=0.015$ in b).
More quantitative methods that have been used to describe mantle mixing include:

1. *Box counting methods.* These are simple statistical approaches to describe the mixing of a small heterogeneity that use a subdivision of the computational domain into a finite number of uniform boxes. By keeping track of the concentration of the heterogeneity in each box and the related statistical distribution we can make quantitative statements about departures of uniformity and mixing times. We can illustrate this by draping a uniform 70x20 cell grid over the convection geometry of Figure 1 and evaluating over time the number of particles in each of these boxes. Figure 7 displays the fraction of boxes that contain at least one particle of both heterogeneities (I and II). By this criterion, the first heterogeneity affects an exponentially growing number of boxes early on and becomes nearly box filling (>80% of boxes have at least one particle in them) at the end of the computation. In contrast, the second heterogeneity remains in only a limited number of boxes until about t=0.015, when it also shows an exponential growth. See Schmalzl et al. (1996) for further illustrations of this technique in mantle convection models.

![Figure 7](image)

*Figure 7. Time evolution of the fraction of boxes that are affected by heterogeneity I and II. In this case we used 7000 particles to discretize the heterogeneities and a uniform grid of 70x20 boxes.*
2. *Strain markers.* The exponential increase of interfacial area in efficient mixing suggests an approach in which one keeps track of length scales by following the stretching and folding of small line segments, which effectively work as strain markers. In practice this is done by following two particles $x_1$ and $x_2$ that were initially close together, say at spacing $dX$. If the mixing is chaotic the length of the segment between the two particles will grow exponentially: $dx = |x_1 - x_2| = dX \exp (\lambda t)$. After a certain time $t = \tau$ we can evaluate the finite-time Luyaponov exponent $\lambda$. If the mixing is chaotic this exponent is positive and the magnitude indicates the mixing time scale. In an ideal situation, one would model the distance between the two particles by allowing for the full stretching and folding of the initial segment, which can be done for example by modeling the line segment with a high resolution marker chain. For computational reasons it is nevertheless common to reinitialize the strain marker after significant deformation or even to ignore any folding by using the distance between the two particles as a proxy for the Luyaponov exponent. Figure 8 illustrates the potential for this approach. See the caption for details. For other examples of the use of approximations to the finite time Luyaponov exponent in mantle mixing studies see Ferrachat and Ricard (1998), Van Keken and Zhong (1999), and Farnetani and Samuel (2003).
Figure 8. Illustration of the use of Lyapunov exponents to describe the efficiency of mantle mixing. a) temperature at t=0. 1000x250 particles pairs are distributed uniformly in the area indicated by the black box. The particles in each pair are offset by a horizontal distance of $10^{-5}$. The small yellow boxes indicate the source regions for heterogeneity I and II. We trace the distance between the particle pairs as they are advected with the flow and plot their distance at $t=0.004$ (b) and $t=0.02$ (c) in their original location. The black regions correspond to areas where the distance between particle pairs became smaller; the colors indicate minor stretching (dark blue) to extensive stretching (dark red).
3. **Two-particle correlation functions.** A statistical approach to quantify the mixing of a heterogeneity that is represented by many particles is to calculate the distances between each pair of particles and compute the cumulative histogram $H(r)$ which is the number of particles pairs that have a distance of less than $r$. The slope of $\log(H)$ vs $\log(r)$ within a particular range of $r$ indicates the spatial dimension of the particle distribution. For example, if in a two-dimensional box calculation the slope is constant and equal to 2 for $r$ approaching the length of the box it is reasonable to assume that the heterogeneity has been mixed in completely. Illustrations of this method are given in Figure 9 and Schmalzl and Hansen (1994).

![Figure 9. Two-particle correlation function $H(r)$ for heterogeneity I and II at time $t=0$, $t=0.004$, ..., $t=0.02$. $H(r)$ is defined as the number of particle pairs that have a distance less than $r$. Each heterogeneity is traced with 100x70 tracers that are at a distance $10^{-3}$ from each other at time $t=0$. The graph for $t=0.004$ is highlighted with black symbols. On the left lines of slope 1 and slope 2 are indicated. The first graph is the same for](image)
both heterogeneities: at distances $r$ less than the size of the heterogeneity the slope is 2, indicated that the heterogeneity is space filling at that scale. The position of the change in slope to horizontal indicates the spatial extent of the heterogeneity. In the early stages of mixing the slope of the correlation function becomes 1 at intermediate scales, indicating linear features. At the end of the mixing calculation the correlation function has slope 1.5 at most scales, indicating that the linear features are becoming less dominant, but that the heterogeneity has not yet become space filling at these scales. Note that the change in mixing style after $t=0.015$ for heterogeneity II is reflected in the change in slope and change of position of the kink in the correlation function.
4. **Viscous dissipation.** The mixing efficiency of a viscous fluid is related to the viscous dissipation $\Phi$ since $\Phi = \tau : \mathbf{D} = 2\mu \mathbf{D} : \mathbf{D}$ where $\mu$ is the dynamic viscosity. Maps of viscous dissipation can therefore be used to qualitatively predict the differences in mixing efficiency between different regions of the mantle, or between different models of mantle convection (Figure 10).

*Figure 10. Viscous dissipation $\Phi$ for the model of Figure 1. Note the high values in the boundary layers and specifically in the 'fixed' points of the flow.*
3.3 Mantle mixing studies

The fluid dynamical approach to mixing provides fundamental tools, but we have to keep in mind that in our effort to understand mantle heterogeneity we have some unique circumstances to account for. First, we seem to require mechanisms to prohibit efficient mixing between ‘reservoirs’. This contrasts strongly with the need to develop efficient mixers in industrial application to guarantee consistency and provide low-cost manufacturing techniques. Second, we have an incomplete knowledge of the distribution of heterogeneity. We only observe the chemical heterogeneity through volcanism at the Earth’s surface and only at a few locations. The dynamical origin of these ‘leaks’ of the mantle may bear directly on the observed mixing properties. Third, the isotopic chemistry that we observe at the Earth’s surface is not only dependent on the distribution of heterogeneity by mantle mixing, but also on the integrated history of radioactive decay and fractionation events. Finally, we observe mantle heterogeneity through the filter of melting. The accompanying chemical and physical differentiation mechanisms may obscure the relationships between chemical differences in the melt and the mantle source heterogeneity.

Quantitative studies of mantle mixing evolved from early laboratory and numerical experiments that linked plate tectonics with convection in the planetary mantle (McKenzie, 1969; Richter, 1973; McKenzie et al., 1974; Richter and Parsons, 1975). The finite strain theory for the development of seismic anisotropy can be used to describe the deformation and subsequent mixing of mantle heterogeneity as well (McKenzie, 1979). It was also shown that mantle convection leads to the relocalization and redistribution of heterogeneity (Richter and Ribe, 1979). Early convection experiments were limited to low convective vigor (low Rayleigh number) due to computational restrictions. Extrapolation from these experiments to Earthlike conditions led to the realization that the typical mixing time for whole mantle convection should be on the order of 0.5-1 Byr (Hoffman and McKenzie, 1985) which strongly suggests some form of layering to retain long-lived heterogeneity. In contrast, dynamical studies of layering in the mantle demonstrated that strict layering at 670 km depth causes the upper mantle to be too cold (Kopitzke, 1979; Spohn and Schubert, 1982; McNamara and Van Keken, 2000). Furthermore, the required radioelement enrichment of the lower mantle causes the lower mantle to be too hot and forms a substantial thermal boundary layer at 670 km depth (Spohn and Schubert, 1982), unless the mantle heat flow is reduced to values significantly below the global average (Richter and McKenzie, 1981).

Several studies explored the ability of a high viscosity lower mantle to slow down mixing and increase the survival time of heterogeneity, ranging from detailed conceptual arguments (Davies, 1984) to simple 1D (Loper, 1985) and 2D numerical simulations (Gurnis and Davies, 1986ab). These models demonstrated the possibility for significantly longer survival times due to the high viscosity of the lower mantle, but this comes at the cost of a much reduced overall convective vigor and too low surface heatflow.

Improved computational ability and better understanding of non-linear systems allowed for fundamental improvements in mantle mixing studies. A detailed study by Olson et al. (1984) used Fourier analysis to demonstrate that the effects of mixing cascade from low to high wavenumbers and demonstrated the importance of stagnation points in providing more efficient deformation. Olson et al. (1984) showed that mixing in
a stationary cell is best described by laminar mixing. The inclusion of time-dependence (e.g., Hofmann and McKenzie, 1985) allows for more efficient mixing. Christensen (1989) demonstrated a further complication: flows that are driven by time-dependent kinematic surface boundary conditions are more likely to maintain poorly mixed islands or to be characterized by laminar mixing than time-dependent dynamic models, where heterogeneities are rapidly strained and follow an exponential, or turbulent mixing law. Kellogg and Stewart (1991) confirmed the exponential stretching behavior in a 12 Fourier mode approximation to the thermal convection equations.

As was demonstrated by several of these early mixing studies, the stretching and folding of heterogeneity leads to a ‘marble cake’ structure, where the distribution of length scales is generally exponential and depends on the mixing efficiency. Kellogg and Turcotte (1990) showed this using a kinematic, but space-filling 2D time-dependent flow based on the Lorenz equations, with a special focus on the thinning of oceanic crust. The predicted distribution of length-scales matches the observed distribution of pyroxenite bands in depleted lherzolite found in several high-temperature peridotites (Allègre and Turcotte, 1986). This supports the suggestion that the pyroxenite originated from subducted oceanic crust, which would provide an intriguing geological constraint on the efficiency of mantle mixing.

Our understanding of mantle mixing has further improved in the last decade particularly due to the ability of numerical models to explore the effects of realistic convective vigor and three-dimensional flows, the incorporation of new methodology to characterize mixing, and the explicit modeling of the geochemical inputs and outputs to the mantle system. Schmalzl and Hansen (1994) demonstrated using 2D isoviscous models at high Rayleigh number that advective stirring leads to mixing on the large scale first and that mixing between convective cells can be quite inefficient, even if the mixing within each cell is very efficient. Schmalzl et al. (1995) explored mixing in a 3D steady-state model. They found that particle motion in the flow is limited by symmetry surfaces imposed by the temperature and velocity field and regular, non-chaotic motion the particles, from which they concluded that time-dependence is probably essential in generating chaotic particle behavior and efficient mixing. The addition of time-dependence in a 3D model at moderate Rayleigh number (Schmalzl et al., 1996) demonstrated that cross-cell mixing is enhanced, but the efficiency appeared to be less than in the case of 2D convection, probably due to the greater stability of the 3D convective pattern.

Three-dimensional convective patterns can be described by a combination of poloidal (divergent and convergent) and toroidal (strike-slip) components. Ferrachat and Ricard (1998) used a kinematic model of convection driven by a surface boundary condition that mimicked a mid-oceanic ridge with a variable length transform fault. The additional toroidal energy caused a significant portion of the model to undergo chaotic mixing, although islands of laminar mixing could persist. These results suggest that it is essential to incorporate the influence of plates on mantle convection in mixing studies. Van Keken and Zhong (1999) further demonstrated the importance of toroidal motion, using a dynamic model of present day mantle convection. In this model, toroidal motion is generated by the influence of the weak zones corresponding to the present-day plate boundaries. Several regions of laminar mixing are present in this model, but in certain areas such as near Papua New Guinea and the western US, significant toroidal motion
caused cork-screw-like particle paths. This allows for large-scale transport between the convective cells and for efficient, chaotic mixing.

The increasing spatial and temporal resolution of numerical models allows for direct testing of geochemical hypotheses by tracking the evolution of particles that carry isotopic and elemental information. Christensen and Hofmann (1994) provided one of the first quantitative models that detailed the recycling of oceanic crust and its influence on the chemical evolution of the mantle. Their models included the formation of basaltic crust and depleted harzburgite upon melting of the peridotite mantle. Upon transformation to eclogite, the oceanic crust ponds at the base of the mantle and is only slowly entrained by thermal convection. The modeled Pb and Nd isotopic evolution after 3.6 Byr provided an adequate fit to the observed MORB and HIMU ranges, which provides a strong quantitative support to the suggestion that oceanic crust recycling is of fundamental importance to the chemistry of mantle plumes (Hofmann and White, 1980; White and Hofmann, 1982). Van Keken and Ballentine (1998; 1999) developed 2D cylindrical models of mantle convection to study the evolution of He isotopes. When comparing models of similar convective vigor as measured by average surface heat flow and plate velocities it was shown that a higher viscosity lower mantle was not capable of preserving heterogeneity over timescales longer than about 1 Byr. Incorporation of phase changes and moderately temperature-dependent rheology similarly couldn’t prevent large scale mixing, except in the case of an extreme, and probably unrealistic, dynamical influence of the endothermic phase change at 670 km depth. Similar results were obtained by Ferrachat and Ricard (2001), Davies (2002), and Stegman et al. (2002).

A distinct class of models that describe the chemical evolution of the Earth are the so-called ‘box models’, in which assumptions are made about the geometry of distinct reservoirs and their interactions. For example, one can assign four distinct reservoirs in the upper mantle, lower mantle, continental crust and atmosphere and develop differential equations that incorporate radiogenic ingrowth, chemical fractionation effects and assumptions about mass transfer between the reservoirs. Successful models reproduce the observed isotopic ratios and/or concentrations. Applications include studies of noble gas evolution (Kellogg and Wasserburg, 1990; O’Nions and Tolstikhin, 1994; Porcelli and Wasserburg, 1995ab; Albarède, 1998), crustal evolution (Jacobsen and Wasserburg, 1979; O’Nions et al., 1979; Jacobson, 1988), mean stirring times of the mantle (Allègre and Lewin, 1995) and lead-isotope systematics (Galer and O’Nions, 1985; Kramers and Tolstikhin, 1994; Paul et al., 2002).

An important advantage of this type of modeling is that the computational cost is much lower than that of full dynamical models, which allows for the efficient exploration of large parameter spaces and for data inversion to obtain optimal parameter choices (e.g., Allègre and Lewin, 1989; Coltice and Ricard, 1999). More complex physics can be integrated into the box model approach. For example, the internal mixing rate in individual reservoirs can be described in parameterized form, as was demonstrated by Kellogg et al. (2002). It is expected that we will see a further integration between the box and fluid dynamical modeling approaches, such as the comparison made by Coltice et al. (2000).
4. Outlook

4.1 Conceptual model development

The improved geochemical and geophysical observations, coupled with enhanced computational ability, has led to an expansion of research on the chemical evolution of the Earth’s mantle. The apparent contradictions between geophysical indications for whole mantle flow and the geochemical requirements for substantial and long-lived heterogeneity have led to a large set of new or revived conceptual models. Proposals include mantle layering below 670 km depth (Kellogg et al., 1999; Anderson, 2002), ‘zoning’ of the mantle due to the variable thermal and chemical properties of subducting slabs (Albarède and Van der Hilst, 2002), chemical variations hidden in small and highly viscous blobs (Becker et al., 1999), and recent breakup of a layered mantle system (Allègre, 2002). For a review of the pros and cons of these various models see e.g., Van Keken et al. (2002).

An alternative model that is gaining rapid popularity incorporates the recycling of oceanic crust in whole mantle convection (e.g., Coltice and Ricard, 1999; Ferrachat and Ricard, 2001; Helffrich and Wood, 2001; Van Keken et al., 2002; Davies, 2002; Carlson and Hauri, 2003). This idea has several attractive aspects. The formation of oceanic crust at mid-oceanic ridges and the subsequent recycling at subduction zones is the foremost modern-day method of introducing heterogeneity into the mantle. The signal of recycled oceanic crust in ocean island lavas has been clearly shown in a number of isotope systems (e.g., Hofmann and White, 1980; White and Hofmann, 1982; Hauri and Hart, 1997; Lassiter and Hauri, 1998; Eiler et al., 2000; Hauri, 2002). The higher density of basalt-derived components in the mantle allows for a natural explanation for long residence times.

Commonly cited problems with this model include the difficulty in explaining the noble gas observations and the distribution of heat producing elements. In particular, the observation of high $^{3}\text{He}/^{4}\text{He}$ in hotspots has been interpreted as the consequence of preservation of a ‘primitive’ or ‘primordial’ component, since $^{3}\text{He}$ is not produced in the Earth’s interior. Yet, the oceanic basalts with high $^{3}\text{He}/^{4}\text{He}$ are non-primitive by any other geochemical measure (e.g., Hart et al., 1992; Hanan and Graham, 1996) and this would suggest that low concentrations of U and Th, and consequently low concentrations of $^{4}\text{He}$ compared to $^{3}\text{He}$ are responsible for the high $^{3}\text{He}/^{4}\text{He}$ ratio. This in turn enables a small $^{3}\text{He}$ residue to account for the high OIB $^{3}\text{He}/^{4}\text{He}$ (Anderson, 1998b; Albarède, 1998; Coltice and Ricard, 1999). Quantitative support for this non-primitive source of high $^{3}\text{He}/^{4}\text{He}$ is provided by a number of models (Ferrachat and Ricard, 2001; Coltice and Ricard, 2002). Other calculations show that because of the residence time of recycled material (1 to 1.5 Byr), U+Th depletion of the protolith is not required if this is mixed with a small volume of $^{3}\text{He}$ rich material (Ballentine et al., 2002). The latter calculations are consistent with radioelement and noble gas concentrations inferred from the Iceland plume (e.g., Hilton et al., 2000).

The concentration of rare gases in the upper mantle can provide additional observational evidence. The $^{3}\text{He}$ concentration in the oceans has been used to argue for a low helium concentration in the upper mantle and a requirement for the deep burial and
isolation of helium (e.g., O’Nions and Oxburgh, 1983). The related low estimates for the $^{40}$Ar concentration in the upper mantle provide a similar argument for argon and potassium storage in the deep mantle. It is interesting to note that these arguments are all dependent on the assumption that the present day concentration of helium in the oceans, which has a residence time of about 1000 years, allows for accurate estimate of upper mantle concentrations. It is not unreasonable to expect that due to variable volcanic activity the present-day $^3$He concentration in the oceans may not be representative of the time-averaged concentrations in the mantle (e.g., Van Keken et al., 2001). In fact, an increase in the noble gas concentration of the upper mantle by a factor of 3.5 removes all requirements for layering and hidden noble gas reservoirs (Ballentine et al., 2002).

An additional argument for mantle layering is based on the mantle concentration of heat producing elements. The extremely depleted nature of N-MORB (Hofmann, 1988) suggests quite low U, Th, and K concentrations in the upper mantle, which would require substantial deeper enrichment (e.g., Kellogg et al., 1999). A new compilation of MORB analysis (Su and Langmuir, 2002) from the PetDB database (Lehnert et al., 2000) suggests that the average oceanic crust is considerably less depleted than N-MORB and that consequently the upper mantle composition is far less depleted than previously assumed. This new analysis effectively removes the requirement for storage of high heat producing element concentrations in the deep mantle (Carlson and Hauri, 2003).

4.2 Quantitative modeling

Any conceptual model requires quantitative support, ideally through an integrated physical and chemical modeling approach. Even if we consider that we have come a long way in developing better geodynamical models, we are still faced with significant hurdles in the development of consistent dynamical models that would allow a full understanding of the dynamical and chemical evolution of the Earth. A number of important concepts that need better resolution include:

- The integration of plate tectonics and mantle convection (e.g., Tackley, 2000a). It is evident that the large-scale plate structure has an important influence on the mantle convection pattern (e.g., Anderson, 1998a). Thus far it has been very difficult to develop models that satisfactorily produce self-consistent, and persistent, plate tectonic style deformation on top of mantle convection, although encouraging progress has been made in recent years (e.g., Trompert and Hansen, 1998; Tackley, 1998, 2000bc, Tackley and Xie, 2002; Bercovici, 2003). This is of particular importance for the testing of conceptual mixing models since otherwise it is difficult, if not impossible, to use realistic temperature-dependent rheology or to accurately differentiate between mid-oceanic ridge and hotspot sampling of the mantle.
- The cause and source regions of hotspots. Since plumes are considered to deliver the signals from the deep mantle it is essential to understand how plumes influence mantle mixing. Of particular concern are the low internal viscosity of mantle plumes and the interaction with the transition zone and lithosphere (e.g., Hauri et al., 1994; Marquart and Schmeling, 2000; Farnetani et al., 2002; Kumagai, 2002).
- The influence of rheological variations on mantle mixing, such as proposed for the survival of primitive blobs (Manga, 1996; Becker et al., 1999) or the mixing of...
oceanic crust (Spence et al., 1988; Van Keken et al., 1996; Merveilleux du Vignaux and Fleitout, 2002). The influence of non-Newtonian rheology on mantle mixing also requires much more detailed investigation (e.g., Ten et al., 1997).

- **Influence of chemical buoyancy due to melt extraction** (Christensen and Hofmann, 1994; Dupeyrat et al., 1995; Davies, 2002). Bulk chemistry changes upon melting are likely of fundamental importance in maintaining heterogeneity since the associated chemical buoyancy changes can effectively counteract thermally driven mixing.

- **The role of the mantle transition zone.** Seismological observations and petrological predictions strongly suggest an important dynamic role for the transition zone (e.g., Ringwood, 1982). This includes the influence of the phase changes, possible variations in minor and major element chemistry, and rheological variations. In particular the 670 km discontinuity has been suggested to function as a partial chemical filter (e.g., Christensen and Yuen, 1984; Weinstein, 1992; Van Keken et al., 1996).

- **Structure and dynamics of the lowermost mantle.** This region includes the D” layer, which is characterized by major chemical and thermal variations. It is likely of fundamental importance to the chemical evolution of the mantle and may function as a (temporary) resting place for subducted slabs. It is also expected to influence the stability of mantle plumes (Davaille et al., 2002; Jellinek and Manga, 2002), the entrainment and residence times of chemical heterogeneity (Olson and Kincaid, 1991; Schott et al., 2002) and the thermal, chemical, and seismological characteristics of compositional variations (Kellogg et al., 1999; Tackley, 2002).

- **Thermal evolution of the Earth.** In particular we need to understand how we can construct chemical models that provide sufficiently realistic cooling histories. It has been pointed out that the relatively low present day heat production in the bulk silicate Earth as compared to the present day heat loss requires some form of buffering or layering (e.g., Butler and Peltier, 2002). We also need to understand the tectonic regime during the Hadean and Archean. The projected higher mantle temperatures in the Archean lead to conditions that are not necessarily conducive to faster plate tectonics. The deeper melting that occurs in a hotter Earth causes more extensive melting and the thicker basalt and depleted peridotite layers provide a strong compositional stabilizing force to plate tectonics. The tectonics of the Archean Earth may well have been dominated by thermal plumes or eclogite driven delamination (e.g., Anderson, 1979; Vlaar et al., 1994; Zegers and Van Keken, 2001).

In conclusion, the renewed interest in the integration of geophysical and geochemical approaches to develop a better understanding of the chemical evolution of the mantle has led to a significant number of new or revived conceptual models. Models that combine geodynamics and geochemistry can provide quantitative tests to these ideas. Although several hurdles still exist, we can expect that the growth of computational resources - combined with better insights into the role of chemical mixing, improved geochemical and seismological observations, and a more fundamental understanding of the interpretation of Earth structure from these observations –will ultimately allow for the
development of a consistent description of the Earth’s mantle evolution as it is influenced by the generation and destruction of chemical heterogeneity.

Acknowledgments
We thank Francis Albarède, Don Anderson, Geoff Davies, Jamie Kellogg, Louise Kellogg, Dan McKenzie, Don Porcelli, and Paul Tackley for stimulating discussions and Rick Carlson for encouragement.
Appendix A

The numerical convection model that is used to illustrate the visualization and quantification of mixing (Figures 1-10) is based on the solution of the equations governing convection in the Earth’s mantle, assuming that the mantle can be described as an anelastic and weakly compressible fluid at infinite Prandtl number. Under the extended Boussinesq approximation we can write the equations of motion as

\[-\nabla P + \nabla \cdot (\eta \dot{\varepsilon}) = RaT \hat{g}\]  

(1)

and the mass conservation equation as

\[\nabla \cdot \mathbf{v} = 0\]  

(2)

where \(P\) is dynamic pressure, \(\eta\) is the viscosity, \(\dot{\varepsilon}\) the strain-rate tensor, \(Ra\) is the Rayleigh number, \(T\) is temperature, and \(\hat{g}\) is the gravity vector. The heat equation incorporates terms that describe viscous heating and adiabatic cooling and heating, and can be written as

\[\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T + \alpha DwT = \nabla^2 T + \frac{Di}{Ra} \sigma_{ij} \frac{\partial u_i}{\partial x_j} - DiwT_0\]  

(3)

where \(t\) is time, \(Di\) is the dissipation number, \(w\) the vertical component of velocity, \(\sigma_{ij}\) the components of the stress tensor, \(u_i\) the velocity components and \(T_0\) is the reference (surface) temperature (e.g., Jarvis and McKenzie, 1980). The equations above are non-dimensional.

The Rayleigh number \(Ra\) and dissipation number \(Di\) are given by

\[Ra = \frac{\rho g \alpha \Delta Th^3}{\eta \kappa}\]  

(4)

and

\[Di = \frac{\alpha \rho g h}{c_p}\]  

(5)

where \(\rho\) is density, \(\alpha\) is expansivity, \(\Delta T\) the temperature contrast across the mantle, \(h\) the depth of the mantle, \(\eta_0\) the reference viscosity, and \(\kappa\) the thermal diffusivity. The parameters on the right hand side of these equations are dimensional.

We use a moderately temperature- and pressure-dependent viscosity which is written in non-dimensional form as

\[\eta(T, p) = exp(-aT + bz)\]  

(6)
where $z$ is depth and $a$ and $b$ non-dimensional coefficients.

The equations (1)-(3) are solved using a finite element approach based on the general tool box Sepran (Cuvelier et al., 1986; http://dutita0.twi.tudelft.nl/sepran/sepran.html) The Stokes equations (1) and incompressibility constraint (2) are solved using a penalty function approach. The heat equation is solved after Galerkin discretization. The time-dependent equations are solved using a second order predictor-corrector method. See Van den Berg et al. (1993) and Van Keken and Ballentine (1999) for more details. The tracers are advanced using a 4th order Runge-Kutta method. The time-step is limited to 50% of the Courant-Friedrichs-Levy criterion. The numerical implementation has been extensively tested against standard mantle convection benchmarks (Blankenbach et al., 1989; Van Keken et al., 1997) as well as published results by other workers.

For the model used in this chapter we use a 2D Cartesian grid with aspect ratio 3.5. The bottom and top boundary are free-slip and the side boundaries are reflective. Temperatures are fixed at top ($T = 0$) and bottom ($T = 1$). We assume whole mantle convection and choose $Ra = 10^6, Di = 0.2, a = -4.60517, b = 3$. There is no internal heating. The numerical grid consists of 2x140x40 quadratic triangles (corresponding to 281x81 nodal points) which provides significantly higher resolution than that needed for the accurate solution of the heat equation. The higher resolution allows for more accurate particle tracking, which is more sensitive to small errors. The initial condition (Figure 1a) was obtained from a previous calculation that was run for sufficiently long time to reach a statistical equilibrium. The mixing calculations are for a time period from 0 to 0.02, which corresponds to a dimensional period of 5 Byr. The convection is moderately time-dependent and the overall convective vigor is low compared to that of the present day Earth. For example, the average surface heatflow is 40 mW/m$^2$ and the average velocity 1 cm/yr. The transit time (time for a particle to travel from top to bottom at average velocity) is 280 Myr, although it should be noted that the velocity in the up- and downwellings is substantially higher.
References


